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Achieving k-barrier Coverage in Hybrid Directional Sensor Networks

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Abstract—Barrier coverage is a critical issue in wireless sensor networks for security applications (e.g., border protection) where directional sensors (e.g., cameras) are becoming more popular than omni-directional scalar sensors (e.g., microphones). However, barrier coverage cannot be guaranteed after initial random deployment of sensors, especially for directional sensors with limited sensing angles. In this paper, we study how to efficiently use mobile sensors to achieve k-barrier coverage. In particular, two problems are studied under two scenarios. First, when only the stationary sensors have been deployed, what is the minimum number of mobile sensors required to form k-barrier coverage? Second, when both the stationary and mobile sensors have been pre-deployed, what is the maximum number of barriers that could be formed? To solve these problems, we introduce a novel concept of weighted barrier graph (WBG) and prove that determining the minimum number of mobile sensors required to form k-barrier coverage is related with finding k vertex-disjoint paths with the minimum total length on the WBG. With this observation, we propose an optimal solution and a greedy solution for each of the two problems. Both analytical and experimental studies demonstrate the effectiveness of the proposed algorithms.

Index Terms—Barrier coverage, mobile, hybrid, directional sensor, wireless sensor networks.

1 Introduction

Wireless sensor networks (WSNs) have been widely used as an effective surveillance tool for security applications, such as battlefield surveillance, border protection, and airport intruder detection. To detect intruders who penetrate the regions of interest (ROI), we need to deploy a set of sensor nodes that can provide coverage of the ROI, a problem that is often referred to as *barrier coverage* [11], where sensors form *barriers* for intruders. A sensor network provides k-barrier coverage for an ROI if all crossing paths through the region is k-covered and a crossing path is said to be k-covered if it can be covered by at least k distinct sensors.

When only stationary sensors are used, however, after the initial random or manual deployment, it is possible that sensors could not form a barrier due to gaps in their coverage, which would allow intruders to cross the ROI without being detected. In fact, it is difficult if possible at all to improve barrier coverage for sensor networks consisting of only stationary sensors. Fortunately, with recent technological advances, practical mobile sensors (e.g., Robomote [6], Packbot [20]) have been developed, which provides us a way to improve barrier coverage performance after sensor networks have been deployed.

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Directional sensors (e.g., camera, radar) have been widely used for security applications. For example, the FREEDOM system [1], deployed on the border between Mexico and the United States, uses cameras to detect illegal intruders. The SBInet project [2], also deployed on the border between Mexico and the United States, uses cameras, radar, and ground sensors to construct a virtual fence to detect illegal intruders. Different from omni-directional scalar sensors, although directional sensors provide extra dimensional information, they usually have limited angle of views and facing directions, which therefore decrease the probability of barrier formation after initial random deployment.

In this paper, we study the barrier coverage formation problem in hybrid directional sensor networks which consist of both stationary and mobile sensors with the directional sensing model. In particular, we consider a two-phase deployment: in the first phase, after stationary sensors are deployed, their barrier gaps are identified and the number of mobile sensors needed can be calculated; in the second phase, mobile sensors are deployed and move to the desired locations to fill in these gaps to form barriers. Figure 1 shows an example of forming a strong barrier using mobile sensors. Mobile sensors 1 and 2 fill in the gaps between stationary sensors and form a strong barrier with pre-existing stationary sensors for the ROI.

A lot of work has been done on barrier coverage.

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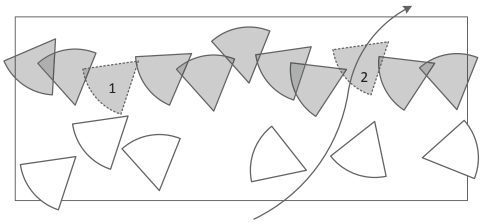

Fig. 1: An example of a strong barrier formation for hybrid directional sensor networks. Mobile sensors 1 and 2 fill in the gaps between stationary sensors and form a strong barrier for the ROI.

However, most of existing work mainly focus on critical condition analysis and barrier construction for stationary sensors with the omni-directional sensing model [5], [11], [15], [18], little effort has been made to explore how to efficiently use mobile sensors to form barrier coverage with stationary sensors, especially for directional sensors. Saipulla et al. [17] used mobile sensors with limited mobility to form a barrier for omni-directional sensors. Our work is different from theirs in the following aspects. First, we study k-barrier coverage formation on directional sensors rather than 1-barrier coverage formation on omni-directional sensors. Second, we want to find the minimum number of mobile sensors needed to form k barriers when only stationary sensors have been deployed. Also, we want to find the maximum number of barriers that could be formed when both the stationary and mobile sensors have been pre-deployed. To the best of our knowledge, we are the first to study how to efficiently form k-barrier coverage in hybrid directional sensor networks.

There are lots of challenging issues in the barrier coverage formation problem of hybrid sensor networks. First, how to determine whether two sensors overlap with each other and calculate the distance between sensors is complicated due to the limited angle of views and variation of facing directions of directional sensors. Second, sensors are usually randomly deployed (dropped by an aircraft), therefore, it is challenging to determine whether the sensors already form k barriers or not after initial deployment. Third, the manufacturing cost of mobile sensors is much higher than that of the stationary sensors [6], which demands the usage of as few mobile sensors as possible. It is therefore challenging to find the minimum number of mobile sensors required to form k-barrier coverage with the deployed stationary sensors. Finally, mobile sensors should move to expected locations to fill in the gaps between stationary sensors. However, sensor movement costs a lot of energy and mobile sensors are often power limited. Therefore, another challenging issue is how to schedule and move mobile sensors to expected locations so that the total moving cost is minimized.

In particular, we study the following problems:

1) ***Min-Num-Mobile(k) problem***: Given an ROI and a deployed sensor network with only stationary sensors, does the network provide k-barrier coverage for the ROI? If not, what is the minimum number of mobile sensors required to form k-barrier coverage with the deployed stationary sensors?
2) ***Max-Num-Barrier problem***: Given an ROI and a deployed sensor network with both stationary and mobile sensors, what is the maximum number of barriers that could be formed?
3) ***Minimum cost barrier formation (MCBF) problem***: After the number of mobile sensors needed is calculated, how to move mobile sensors to fill in the gaps to form barriers so that the total moving cost is minimized?

In this paper, we systematically address the aforementioned problems, and the main contributions of this paper are summarized as follows:

- To the best of our knowledge, we are the first to study the barrier coverage formation problem in hybrid directional sensor networks with both stationary and mobile sensors.
- We introduce a weighted barrier graph (WBG) model for the study of the barrier coverage formation problem. We prove that determining the minimum number of mobile sensors required to form k-barrier coverage is related with finding k vertex(sensor)-disjoint [1] paths with the minimum total length on the WBG.
- We propose optimal solutions and efficient greedy solutions for the *Min-Num-Mobile(k) problem* and the *Max-Num-Barrier problem*.
- We formulate the problem of relocating mobile sensors to form k-barrier coverage while minimizing the total moving cost as a minimum cost bipartite assignment problem, and solve it in polynomial time using the Hungarian algorithm.

The remainder of the paper is organized as follows. We give a brief discussion about the literature of barrier coverage in Section 2. We present the system model in Section 3. We introduce the WBG and present theoretical analysis of the barrier coverage formation problem for directional sensor networks in Section 4. We elaborate on the optimal and the greedy solution for the *Min-Num-Mobile(k) problem* and the *Max-Num-Barrier problem* in Section 5 and Section 6. We present the solution to the MCBF problem in Section 7. The performance evaluation is presented in Section 8. Section 9 discusses the system model and the proposed algorithms. Finally, we conclude the paper in Section 10.

2 Related Work

Kumar et al. [11] firstly defined the notion of k-barrier coverage for WSNs and proposed an efficient

1. Without confusion, we interchangeably use *vertex-disjoint* and *sensor-disjoint* throughout this paper.

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algorithm to determine whether a belt region is k-barrier covered or not. They also introduced two notions of probabilistic barrier coverage - weak barrier coverage and strong barrier coverage, and derived critical conditions for weak k-barrier coverage in randomly deployed sensor networks. Kumar et al. [12] further proposed a centralized, optimal sleep-wakeup algorithm to prolong the lifetime of barrier coverage. Chen et al. [5] introduced the notion of local barrier coverage and devised localized sleep-wakeup algorithms that provide near-optimal solutions. Liu et al. [15] devised an efficient distributed algorithm to construct multiple disjoint barriers for strong barrier coverage in a randomly deployed sensor network on a long irregular strip region. Saipulla et al. [18] studied the barrier coverage of the line-based deployment rather than the Poisson distribution model, and a tight lower-bound for the existence of barrier coverage was established. Li et al. [14] studied the weak k-barrier coverage and derived a lower bound for the probability of weak k-barrier coverage with and without considering the border effect, respectively.

Recently, barrier coverage in directional sensor networks has gradually received more and more attention. Zhang et al. [25] studied the strong barrier coverage problem for rotationally directional sensors. A novel full-view coverage model was introduced in [23] for camera sensor networks. A full-view coverage verification method was proposed and an estimate of deployment density to achieve full-view coverage for the whole monitored area was given. With the full-view coverage model, Wang et al. [22] further proposed a novel method to select camera sensors from an arbitrary deployment to form a camera barrier. Directional sensor arrays were built to form a barrier to detect and localize intruders in [24]. The minimum camera barrier coverage problem was studied in camera sensor networks [16]. Tao et al. [21] investigated the problem of finding appropriate orientations of directional sensors such that they can provide strong barrier coverage.

With the development of mobile sensors, node mobility is exploited to improve barrier coverage. Shen et al. [19] studied the energy efficient relocation problem for barrier coverage with mobile sensors. A centralized barrier algorithm was proposed to compute the relocated positions for all sensors to form a barrier. Keung et al. [9] focused on providing k-barrier coverage against moving intruders. They demonstrated that the problem is similar to classical kinetic theory of gas molecules in physics, and derived the inherent relationship between barrier coverage and a set of crucial system parameters including sensor density, sensor and intruder density. Ban et al. [3] studied the problem on how to relocate mobile sensors to construct k grid barriers with minimum energy consumption. He et al. [8] studied the cost-effective barrier coverage problem when there are not sufficient mobile sensors and designed sensor patrolling algorithms to improve barrier coverage. Saipulla et al. [17] proposed a greedy algorithm to find barrier gaps and moved mobile sensors with limited mobility to improve barrier coverage.

3 System Model and Preliminaries

In this section, we present the system model including the network model and the sensing model for directional sensors, and introduce some preliminaries about barrier coverage and directional sensors.

3.1 System Model

We assume that the ROI is a two-dimensional rectangular belt area. For the *Min-Num-Mobile(k) problem*, n stationary sensors are randomly deployed in the belt region. For the *Max-Num-Barrier problem*, n stationary sensors and τ mobile sensors are randomly deployed in the belt region. We assume that they are the same type of sensors except that mobile sensors have the ability to move. Let $S = \{s_1, s_2, \cdots, s_n\}$ denote the set of stationary sensors.

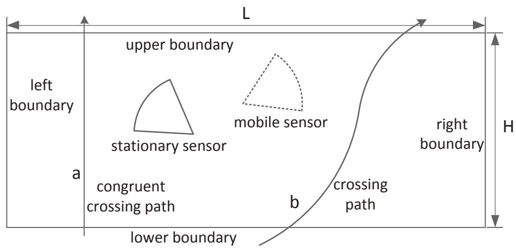

Fig. 2: An illustration of belt region (the square area), crossing paths, and directional sensors.

As shown in Figure 2, the area with the length of L and the width of H is generally a long and thin strip. A crossing path is a path that crosses the complete width of the area from the lower boundary to the upper boundary. A congruent crossing path is a crossing path that is orthogonal to the two boundaries. The path a and path b shown in Figure 2 demonstrate a congruent crossing path and a random crossing path, respectively. An intruder may attempt to penetrate the area along any crossing path.

Unlike an omni-directional sensor, a directional sensor has a limited angle of view and an orientation. Therefore, as shown in Figure 3(a), a sector is commonly adopted to represent the sensing model of directional sensors. Let s_i denote the directional sensor i, then it can be represented by a 5-tuple $< x_i, y_i, r, \alpha, \beta_i >$, where $l_i = (x_i, y_i)$ is the two-dimensional location of the center of sensor i, r is the sensing range and α is half of the sensing angle of a sensor. We assume that each sensor has the identical sensing range and sensing angle. Relaxations to this assumption will be discussed in Section 9. According to the ground truth data in [7], the sensing angle of

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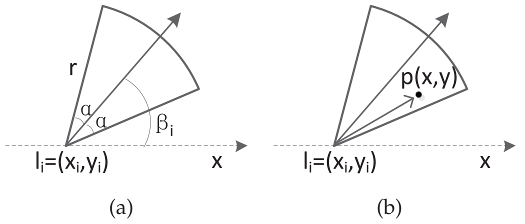

Fig. 3: (a) The sector sensing model for directional sensors; (b) A point p is covered by the sensor s_i.

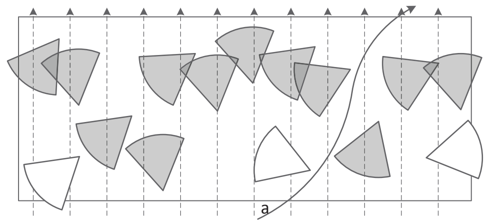

Fig. 4: An example of weak barrier coverage formed by sensors in grey color.

directional sensors, 2α, is usually less than π. β_i is the orientation or the facing direction of sensor i. We assume that β_i is uniformly distributed in $[0, 2\pi)$, e.g., $\beta_i \sim U(0, 2\pi)$. Note that the omni-directional sensing model is a special case of the directional sensing model when $2\alpha = 2\pi$.

Definition 1. *A two-dimensional point $p = (x, y)$ is said to be covered by a directional sensor $s_i = <x_i, y_i, r, \alpha, \beta_i>$ if and only if the following two conditions are satisfied.*

- $(x - x_i)^2 + (y - y_i)^2 \leq r^2$,
- $ang(\overrightarrow{l_i p}) \in [\beta_i - \alpha, \beta_i + \alpha]$, *where $ang(\cdot)$ denotes the angle of $(\cdot)$.*

The largest coverage range of a directional sensor, denoted by l_r, is the length of the longest line in its sensing sector. Since the longest line is either the sensing radius or the longest chord of the sector, we have

$$l_r = \begin{cases} \max\{r, 2r \sin\alpha\} & 0 \leq \alpha < \frac{\pi}{2}, \\ 2r & \frac{\pi}{2} \leq \alpha \leq \pi. \end{cases} \tag{1}$$

3.2 Preliminaries

Kumar et al. proved that a network provides k-barrier coverage if and only if there exists k sensor-disjoint barriers in the ROI [11]. The term of sensor-disjoint barriers means that none of any two barriers have sensors in common. Therefore, in order to provide k-barrier coverage for the ROI, mobile sensors should form k sensor-disjoint barriers with the stationary sensors.

Two types of barrier coverage: *weak barrier coverage* and *strong barrier coverage*, were also introduced in [11]. Weak barrier coverage requires that the union of sensors form a barrier in the horizontal direction from the left boundary to the right boundary, so that every intruder moving along congruent crossing paths can be detected. Figure 4 shows an example of weak barrier coverage. However, weak barrier coverage cannot guarantee the detection of intruders following any crossing path (e.g., path a). In contrast, strong barrier coverage requires that the union of sensors forms a barrier from the left boundary to the right boundary so that any intruder can be detected no matter what crossing path it takes. An example of strong barrier coverage is shown in Figure 1. In this paper, we address the barrier coverage formation problem for both weak and strong barrier coverage.

The fundamental problem for weak barrier coverage is to decide whether two directional sensors overlap in the horizontal direction or not. Let x_i^L and x_i^R denote the left and the right coverage boundary of sensor s_i in the horizontal direction, which can be obtained by geometric calculation.

Definition 2. *Directional sensors s_i and s_j are said to be* ***weakly connected*** *if they overlap in the horizontal direction, that is, $x_i^L \leq x_j^L \leq x_i^R$ or $x_j^L \leq x_i^L \leq x_j^R$.*

Definition 3. *Directional sensors s_i and s_j are said to be* ***strongly connected*** *if they overlap with each other.*

The problem to decide whether two sensors overlap with each other is easy to answer for omni-directional sensors of disk sensing model. However, it is much harder for directional sensors due to their different orientations and limited angle of views. For example, we can claim that two omni-directional sensors overlap with each other if the Euclidean distance between their centers is smaller than or equal to $2r$. However, two directional sensors might not overlap even when they are very close to each other, e.g., two cameras can be side by side but looking at opposite directions. Therefore, using only distance information would not work for directional sensors. Note that the sensing region of a directional sensor is bounded by two line segments and an arc. We have the following Lemma.

Lemma 1. *Directional sensors s_i and s_j overlap with each other if and only if there exists at least one intersection between the two line segments and the arc of s_i and the two line segments and the arc of s_j.*

Proof: $\Rightarrow$. If there exists an intersection between the two line segments and the arc of s_i and the two line segments and the arc of s_j, there must exist one point covered by both s_i and s_j. Then s_i and s_j overlap with each other.

$\Leftarrow$. If s_i and s_j overlap with each other, there exists at least one point covered by both s_i and s_j. Since the point is bounded by the two line segments and the arc of each sensor, there must exist at least one intersection between the two line segments and the arc of s_i and that of s_j. □

Based on Lemma 1, the problem of deciding whether s_i and s_j overlap or not can be simplified

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to check whether there exist intersections between the line segments of s_i and the line segments of s_j, the line segments of s_i (s_j) and the arc of s_j (s_i), and the arc of s_i and the arc of s_j.

Table 1 summarizes the notations used in the paper.

TABLE 1: Summary of Notations

Symbol	Description
L	the length of the belt region
H	the width of the belt region
n	the number of stationary sensors deployed
τ	the number of mobile sensors deployed for the *Max-Num-Barrier problem*
s_i	the ith stationary sensor
l_i	$l_i = (x_i, y_i)$ the location of s_i
r	the sensing range
α	half of the sensing angle
β_i	the facing direction of s_i
l_r	the largest coverage range of each sensor
G	the weighted barrier graph $G = (V, E, W)$
P_q^*	the set of q vertex-disjoint paths with the minimum total length on G
P_q^k	the k-auxiliary set of P_q^*, which is composed of P_q^* and $k - q$ direct paths
$\hat{P}_k$	the optimal set of k sensor-disjoint barriers to the *Min-Num-Mobile(k) problem*
N_m	the minimum number of mobile sensors required for the *Min-Num-Mobile(k) problem*
N_b	the maximum number of barriers for the *Max-Num-Barrier problem*

4 Problem Formulation and Analysis

In this section, we introduce a novel graph model, weighted barrier graph (WBG), to formulate the barrier coverage formation problem for hybrid directional sensor networks, and then present theoretical analysis of the barrier coverage formation problem based on the WBG.

4.1 Weighted Barrier Graph (WBG)

Definition 4. *A* **weighted barrier graph** $G = (V, E, W)$ *of a sensor network is constructed as follows. The set V consists of vertices corresponding to the left boundary (s), all the stationary sensors (S) and the right boundary (t) of the belt region, that is, $V = \{v_1, v_2, \cdots, v_{n+2}\} = \{s \cup S \cup t\}$. $E = \{e(v_i, v_j)\}$ is the set of edges between any pair of vertices except (s, t). $W : E \to \mathbb{R}$ is the set of weights of each edge, where the weight $w(v_i, v_j)$ of edge $e(v_i, v_j)$ is the minimum number of mobile sensors needed to connect v_i and v_j.*

To calculate the minimum number of mobile sensors needed to connect vertices v_i and v_j, the distance between two vertices must be calculated first. Therefore, we further give the following definitions.

Definition 5. Weak distance $d_w(v_i, v_j)$*: the minimum distance between two vertices v_i and v_j in the horizontal direction.*

When both v_i and v_j are stationary sensors, $d_w(v_i, v_j) = 0$ if they are weakly connected; otherwise, $d_w(v_i, v_j) = x_j^L - x_i^R$ given the assumption that $x_j^L > x_i^R$. When v_i is the left boundary s, $d_w(v_i, v_j) = 0$ if v_j intersects with the left boundary; otherwise, $d_w(v_i, v_j) = v_j^L$. When v_i is the right boundary t, $d_w(v_i, v_j) = 0$ if v_j intersects with the right boundary; otherwise, $d_w(v_i, v_j) = L - v_j^R$.

Definition 6. Strong distance $d_s(v_i, v_j)$*: the minimum distance between two vertices v_i and v_j.*

$$d_s(v_i, v_j) = \begin{cases} d_w(v_i, v_j) & \text{if } v_i \text{ or } v_j \text{ is } s \text{ or } t \\ 0 & \text{if } v_i, v_j \in S \text{ and overlap} \\ \min(d(p_i, p_j)) & \text{otherwise} \end{cases} \tag{2}$$

where p_i and p_j are points on the sensing region of v_i and v_j, respectively, $d(p_i, p_j)$ is the Euclidean distance between p_i and p_j.

The minimum number of mobile sensors needed to connect vertices v_i and v_j is, therefore, calculated as follows:

$$w(v_i, v_j) = \begin{cases} \lceil \frac{d_w(v_i, v_j)}{l_r} \rceil & \text{weak barrier coverage} \\ \lceil \frac{d_s(v_i, v_j)}{l_r} \rceil & \text{strong barrier coverage} \end{cases} \tag{3}$$

where l_r is the largest coverage range of a sensor. Note that the weak distance $d_w(v_i, v_j)$ and the strong distance $d_s(v_i, v_j)$ are used for the WBG of weak barrier coverage and strong barrier coverage, respectively.

Figure 5(b) and 5(c) demonstrate the WBG of weak barrier coverage and strong barrier coverage for the sensor network shown in Figure 5(a). Any pair of vertices is connected by an edge except s and t. $w(s, a) = 0$ because sensor a intersects with the left boundary. The two graphs have the same set of vertices and edges but have different set of weights. For example, $w(b, f) = 1$ for weak barrier coverage while $w(b, f) = 2$ for strong barrier coverage, which means that 1 and 2 mobile sensors can weakly or strongly connect sensors b and f, respectively.

4.2 Theoretical Analysis

In the following, we present theoretical analysis of the barrier coverage formation problem based on the WBG. Note that all the conclusions work for both weak and strong barrier coverage.

Lemma 2. *Any path from s to t on the WBG is a barrier composed of pre-existing stationary sensors and virtual mobile sensors. The length of the path is the minimum number of mobile sensors required to form the barrier.*

Proof: According to the definition of WBG, if we choose a path from s to t, and put exactly the number of mobile sensors on each edge of path, then the stationary sensors on the path are connected by mobile sensors, therefore, a barrier is formed. The minimum number of mobile sensors required to form the barrier is equivalent to the sum of weights of all edges on the path, which is the length of the path. □

To better explain Lemma 2, take paths in Figure 5 for example. Suppose we choose the path $s \to a \to$

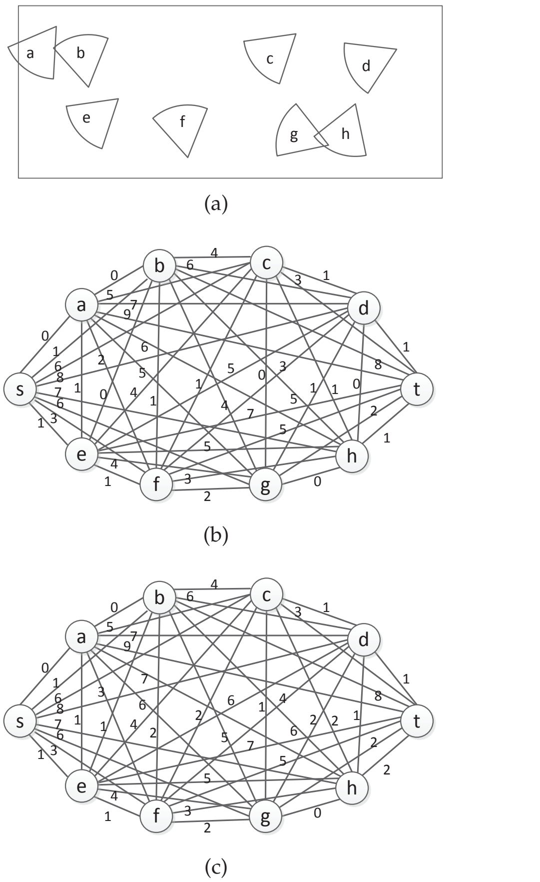

Fig. 5: (a)The deployed sensor network; (b) weighted barrier graph of weak barrier coverage; (c) weighted barrier coverage of strong barrier coverage.

$b \to c \to d \to t$ in Figure 5(b). The path length is 6, which means 6 mobile sensors are required to form the barrier along the path. There are three gaps on the path: $b \to c$, $c \to d$, and $d \to t$, which requires 4, 1 and 1 mobile sensors to fill in them, respectively.

Recall that k-barrier coverage is equivalent to k sensor-disjoint barriers. Based on Lemma 2, we have the following theorem.

Theorem 3. *If each of the k sensor-disjoint barriers to be formed must contain at least one stationary sensor, determining the minimum number of mobile sensors required to form k-barrier coverage with pre-existing stationary sensors is equivalent to finding k vertex-disjoint paths on the WBG with the minimum total length.*

Proof: Based on Lemma 2, each barrier containing at least one stationary sensor must be a path from s to t on the WBG. Therefore, finding k sensor-disjoint barriers is equivalent to finding k vertex-disjoint paths on the WBG. Since we want to use the minimum number of mobile sensors to form k sensor-disjoint barriers, we should find the set of k vertex-disjoint paths on the WBG that has the minimum total length. □

Corollary 4. *The sensor network provides k-barrier coverage for the ROI after initial deployment iff there exist at least k vertex-disjoint paths with length of 0 on the WBG.*

Proof: A path with length 0 on the WBG means the stationary sensors on the path can form a barrier after initial deployment. When no mobile sensors is needed, finding k-barrier coverage is equivalent to finding k vertex-disjoint paths on the WBG. Therefore, a region is k-barrier covered after initial deployment is equivalent to the existence of k vertex-disjoint paths with length of 0 on the WBG. □

Besides all the paths from s to t on the WBG, there is a kind of special paths using only mobile sensors to form barriers. That is, s and t are directly connected by using only mobile sensors. For this kind of barriers, the optimal way of using the minimum number of mobile sensors, obviously, is to deploy them continuously along the horizontal direction. We call this kind of barrier as *direct barrier* and the corresponding path (s,t) as *direct path*. Given the length of belt region is L, the minimum number of mobile sensors needed to form a direct barrier is $\lceil \frac{L}{l_r} \rceil$. We can observe that a direct barrier is always sensor-disjoint from other paths on the WBG, and different direct barriers are always sensor-disjoint from each other. With this observation, we have the following lemma.

Lemma 5. *Given a belt region with length L, the minimum number of mobile sensors required for each barrier in the optimal set to the* Min-Num-Mobile(k) problem *is upper bounded by* $\lceil \frac{L}{l_r} \rceil$.

Proof: Suppose $\hat{P}_k$ is the optimal set of k sensor-disjoint barriers with the minimum number of mobile sensors needed to form k-barrier coverage. If any barrier in $\hat{P}_k$ needs more mobile sensors than a direct barrier, we can always replace it with a direct barrier for less number of mobile sensors needed. Therefore, the previous $\hat{P}_k$ is not the optimal set, which contradicts to our assumption. Hence, no barrier in the optimal set needs more than $\lceil \frac{L}{l_r} \rceil$ mobile sensors. □

Direct barriers are also needed when the vertex-disjoint paths found on the WBG are not enough. Suppose the application requires 5-barrier coverage, but the maximum number of vertex-disjoint paths found on the WBG is 3, then we can add two direct barriers to reach 5-barrier coverage.

Suppose there exist k vertex-disjoint paths on the WBG, and P_k^* denote the set of k vertex-disjoint paths with the minimum total length on the WBG. Note that P_k^* may not the optimal set to the *Min-Num-Mobile(k) problem* that has the minimum total length after considering direct paths, even no path in P_k^* is longer than $\lceil \frac{L}{l_r} \rceil$. We will present the algorithm to find the optimal set in Section 5.

5 THE MIN-NUM-MOBILE(K) PROBLEM

In this section, we present an efficient optimal algorithm and a greedy algorithm to solve the *Min-Num-*

Mobile(k) problem.

Before introducing our optimal algorithm, we first introduce the *vertex-disjoint path algorithm* [4] which can find a set of vertex-disjoint paths with the minimum total length, P_q^*, on a graph.

Given P_i^*, the *vertex-disjoint path algorithm* performs the following steps to find P_{i+1}^*.

Step 1: Graph transformation. Transform the graph G into a new graph NG based on P_i^* by using the following procedures. First, replace the edges of the disjoint paths in P_i^* by arcs directed towards the source, and make the length of the arcs negative; Second, split each vertex (except for endpoint vertices) on the disjoint paths into two co-located subvertices joined by an arc of length zero. Direct the arc of length zero towards the source. Replace each external edge connected to a vertex on the shortest paths by its two arcs of the same length, where one arc is directed to the first subvertex and the other one is directed from the second subvertex.

Step 2: Shortest path finder. Find the shortest path np on the new graph NG using the modified Dijkstra algorithm [4].

Step 3: Path update. Update P_i^* and np to get P_{i+1}^*: transform to the original graph G and erase any edge of this shortest path interlacing with the previous set of vertex-disjoint paths P_i^*. Find the new set of vertex-disjoint paths P_{i+1}^* after removing the interlacing edges.

The initialization of the algorithm is P_1^* which is the shortest path on the graph. Once P_1^* is obtained, we can perform these steps iteratively to find P_2^*, P_3^* and so on. Note that for $i < j$, P_i^* may not be a subset of P_j^*. Take Figure 5(c) as an example, $P_2^* = \{\{s,a,b,c,d,t\}, \{s,e,f,g,h,t\}\}$, and $P_3^* = \{\{s,a,b,c,d,t\}, \{s,e,t\}, \{s,f,g,h,t\}\}$. More details of the algorithm can be found in [4].

5.1 Optimal Algorithm

Let $\hat{P}_k$ denote the optimal set of k sensor-disjoint barriers requiring the minimum number of mobile sensors, and $N_m = |\hat{P}_k|$ denote the minimum number of mobile sensors needed. Note that $|\cdot|$ denotes the total length of paths in $\cdot$. We first define the k-auxiliary set to help us find the optimal set $\hat{P}_k$.

Definition 7. k-*auxiliary set*: *P_q^k is called the k-auxiliary set of P_q^* $(0 \le q \le k)$, which is composed of P_q^* and $k-q$ direct barriers (s,t).*

We leverage the *vertex-disjoint path algorithm* to help the design of our optimal algorithm. The basic idea is to first find all the sets of vertex-disjoint paths with the minimum total length on the WBG, and then extend each set P_q^* $(0 \le q \le \eta)$ to its k-auxiliary set P_q^k, where $\eta = \min(k,\zeta)$ and ζ is the maximum number of vertex-disjoint paths on the WBG. The optimal set $\hat{P}_k$ is the k-auxiliary set that has the minimum total length among all k-auxiliary sets.

Algorithm 1 Min-Num-Mobile(k)-Optimal algorithm

Input: Weighted barrier graph G, L, k and l_r
Output: $\hat{P}_k$ and N_m
1: Let $P_0^* \leftarrow \emptyset$ and P_0^k denote a set of k direct barriers
2: $P_1^* \leftarrow$ Dijkstra(G)
3: $\eta \leftarrow 1$
4: **while** $\eta < k$ **do**
5: $\quad NG \leftarrow$ graph-transform(G, P_η^*)
6: $\quad$ **if** there exist paths from s to t on NG **then**
7: $\quad\quad np \leftarrow$ modified-Dijkstra(NG)
8: $\quad\quad \eta \leftarrow \eta + 1$
9: $\quad\quad P_\eta^* \leftarrow$ path-update$(P_{\eta-1}^*, np)$
10: $\quad$ **else**
11: $\quad\quad$ break
12: $\hat{P}_k \leftarrow P_\eta^k$, and $N_m \leftarrow |P_\eta^*| + (k-\eta)\lceil \frac{L}{l_r} \rceil$
13: **for** $q = 0$ to $\eta - 1$ **do**
14: $\quad$ **if** $|P_q^*| + (k-q)\lceil \frac{L}{l_r} \rceil < N_m$ **then**
15: $\quad\quad \hat{P}_k \leftarrow P_q^k$, and $N_m \leftarrow |P_q^*| + (k-q)\lceil \frac{L}{l_r} \rceil$

Algorithm 1 describes the details of the optimal algorithm where Step 2 finds the first shortest path on the WBG, i.e., P_1^*, Step 4 through 11 perform the *vertex-disjoint path algorithm* iteratively to find all P_q^* for $1 < q \le \eta$, and Step 13 through 15 find the k-auxiliary set with the minimum total length among all k-auxiliary sets and claim it as the optimal set.

Theorem 6. *The optimal set of k sensor-disjoint barriers requiring the minimum number of mobile sensors, $\hat{P}_k$, is the k-auxiliary set with the minimum total length among all k-auxiliary sets.*

Proof: We first prove that $\hat{P}_k$ must be a k-auxiliary set P_q^k composed of P_q^* $(q \in [0,\eta])$ and $k-q$ direct barriers, where $\eta = \min(k,\zeta)$ and ζ is the maximum number of vertex-disjoint paths on the WBG.

Each barrier either contains at least one stationary sensor or no stationary sensor. Therefore, each barrier in $\hat{P}_k$ is either a path on the WBG or a direct barrier.

Suppose no barrier in $\hat{P}_k$ is a direct barrier, then all barriers in $\hat{P}_k$ are paths on the WBG. We know that P_k^* is the set of k vertex-disjoint paths with the minimum total length on the WBG. Therefore, $\hat{P}_k = P_k^*$, which is composed of P_k^* and $k-k=0$ direct barriers.

When $\hat{P}_k$ contains direct barriers, suppose there are $k-q$ $(0 \le q \le \eta)$ direct barriers in $\hat{P}_k$. We prove that the rest q sensor-disjoint barriers in the optimal set must be P_q^*. We prove it by contradiction. Suppose the rest q sensor-disjoint barriers (vertex-disjoint paths) in $\hat{P}_k$ is not P_q^*, we can always use P_q^* to replace these q sensor-disjoint barriers to get a new set of k sensor-disjoint barriers with smaller total length, which means that $\hat{P}_k$ is not the optimal set. This contradicts to our assumption. Therefore, the rest q sensor-disjoint barriers in $\hat{P}_k$ must be P_q^*.

Therefore, the optimal set of k sensor-disjoint barriers must be composed of P_q^* $(q \in [0,\eta])$ and $k-q$ direct barriers, which is a k-auxiliary set. The total length of a k-auxiliary set is $|P_q^*| + (k-q)\lceil \frac{L}{l_r} \rceil$. Since q ranges from 0 to η, the k-auxiliary set with the minimum total length is the optimal set of k sensor-disjoint barriers and the minimum number of mobile sensors needed is:

$$N_m = \min\{|P_q^*| + (k-q)\lceil \frac{L}{l_r} \rceil\}_{q=0}^{\eta}$$

The optimality of Algorithm 1 is proved. □

Theorem 7. *Given a sensor network with n stationary sensors, the optimal algorithm can solve the* Min-Num-mobile(k) problem *in $O(kn^2)$.*

Proof: The number of vertices on the WBG is $n+2$, which is on the order of n. The number of edges on the graph is $n(n-1)/2-1$, which is on the order of n^2. The *vertex-disjoint path algorithm* consists of graph transformation, modified Dijkstra algorithm and path update. The running time of graph transformation and path update is $O(n)$ and the running time of the modified Dijkstra algorithm is $O(n \log n + n^2)$. Thus, the running time of the vertex-disjoint path algorithm is $O(n^2)$. Since the *vertex-disjoint path algorithm* is performed at most k times, the optimal algorithm can solve the *Min-Num-Mobile(k) problem* in $O(kn^2)$. □

5.2 Greedy Algorithm

The *vertex-disjoint path algorithm* involves a lot of operations, such as graph transformation (node-split and node-merge), which are complicated especially for large-scale networks. In this section, we propose a greedy algorithm which is faster than the optimal algorithm.

The basic idea of the greedy algorithm is to repeatedly find the shortest path on the WBG until k paths are found or the latest found path is longer than $\lceil \frac{L}{l_r} \rceil$ or no path can be found. If, in the end, the number of found paths is smaller than k, additional direct barriers are added to form the k barriers. The procedures of the greedy algorithm are described as follows:

1) Initialize $\hat{P}_k$ as an empty set.
2) If there exist paths from s to t on the WBG, find the shortest path using Dijkstra's algorithm; otherwise, go to 5).
3) If the found shortest path is longer than $\lceil \frac{L}{l_r} \rceil$, discard the path, go to 5); otherwise, go to 4).
4) Add the path into $\hat{P}_k$. If the path is the k-th found path, stop; otherwise, remove all the vertices (except s and t) on the found path from the WBG, go to 2).
5) Suppose the number of paths in $\hat{P}_k$ is q, add $k-q$ direct barriers into $\hat{P}_k$.

The pseudocode of the greedy algorithm is presented in Algorithm 2.

Theorem 8. *Given a sensor network with n stationary sensors, the greedy algorithm can solve the* Min-Num-Mobile(k) problem *in $O(kn^2)$.*

Algorithm 2 Min-Num-Mobile(k)-Greedy algorithm

Input: Weighted barrier graph G, L, k and l_r
Output: $\hat{P}_k$ and N_m
$\hat{P}_k \leftarrow \emptyset$, $q \leftarrow 0$
while $q < k$ **do**
  if there exist paths from s to t on G **then**
    $p \leftarrow$ Dijkstra(G)
    if $|p| \leq \lceil \frac{L}{l_r} \rceil$ **then**
      $\hat{P}_k \leftarrow \hat{P}_k \cup p$
      $q \leftarrow q+1$
      update G by removing all the vertices (except s and t) on p
  else
    break;
$N_m \leftarrow |\hat{P}_k|$
if $q < k$ **then**
  $\hat{P}_k \leftarrow \hat{P}_k \cup \underbrace{\{(s,t), \cdots, (s,t)\}}_{k-q}$, and $N_m \leftarrow |\hat{P}_k|$

Proof: We have shown that the number of vertices and edges on the WBG are on the order of n and n^2, respectively. Therefore, the running time of Dijkstra's algorithm is $O(n^2)$. Since the greedy algorithm runs Dijkstra's algorithm at most k rounds, the greedy algorithm can solve the *Min-Num-Mobile(k) problem* in $O(kn^2)$. □

Although the running times of the optimal algorithm and the greedy algorithm are both $O(kn^2)$ in the worst case, the greedy algorithm is usually much faster than the optimal algorithm, especially for large scale networks, since it does not need to perform graph transformation and path update. We will show the comparison of computation time between two algorithms in the performance evaluation section.

6 The Max-Num-Barrier problem

Once the minimum number of mobile sensors required to form k-barrier coverage is solved, the *Max-Num-Barrier problem* can be solved accordingly. Notice that the *Max-Num-Barrier problem* is studied under a different scenario where both the stationary and mobile sensors have been pre-deployed.

Given an ROI and a deployed hybrid sensor network with n stationary and τ mobile sensors, the maximum number of barriers that could be formed, denoted by N_b, is k if the minimum number of mobile sensors required to form k-barrier coverage is less than or equal to τ, but the minimum number of mobile sensors required to form $(k+1)$-barrier coverage is larger than τ. Therefore, the optimal solution to the *Max-Num-Barrier problem* is based on the optimal solution to the *Min-Num-Mobile(k) problem*. In the following, we propose an optimal algorithm as well as a faster greedy algorithm to solve the *Max-Num-Barrier problem*.

6.1 Optimal Algorithm

The optimal algorithm is described as follows:

1) Perform Algorithm 1 (the *Min-Num-Mobile(k)-Optimal Algorithm*) with k increasing until $|\hat{P}_{k+1}| > \tau$.
2) The maximal number of barriers is k.

According to Theorem 6, the set of N_b barriers is composed of a set of vertex-disjoint paths P_q^* on the WBG and direct barriers. Therefore, we have

$$N_b = q + \lfloor (\tau - |P_q^*|)/\lceil \frac{L}{l_r} \rceil \rfloor \tag{4}$$

Theorem 9. *The maximum number of barriers N_b is lower bounded by $\lfloor \tau / \lceil \frac{L}{l_r} \rceil \rfloor$ and upper bounded by $n + \lfloor \tau / \lceil \frac{L}{l_r} \rceil \rfloor$.*

Proof: When all barriers are direct barriers, the maximum number of barriers reaches its lower bound $\lfloor \tau / \lceil \frac{L}{l_r} \rceil \rfloor$. In Eq. 4, $N_b = q + \lfloor (\tau - |P_q^*|)/\lceil \frac{L}{l_r} \rceil \rfloor$. For a WBG, $q \le n$ because the maximum number of vertex-disjoint paths on it cannot be larger than the number of stationary sensors n. When q reaches n, and the total length $|P_n^*|$ is 0, the maximum number of barriers reaches its upper bound $n + \lfloor \tau / \lceil \frac{L}{l_r} \rceil \rfloor$. □

Theorem 10. *For a deployed sensor network with n stationary sensors and τ mobile sensors, the optimal algorithm can solve the* Max-Num-Barrier problem *in $O(n^3)$.*

Proof: The basis of the optimal algorithm is the *Min-Num-Mobile(k)-Optimal Algorithm*, the running time of which is $O(kn^2)$ for k barriers. According to Theorem 9, in the worst case, the maximum number of barriers could be $n + \lfloor \tau / \lceil \frac{L}{l_r} \rceil \rfloor$. The running time of the *Min-Num-Mobile(k)-Optimal Algorithm* is $O(n^3 + n^2 \lfloor \tau / \lceil \frac{L}{l_r} \rceil \rfloor)$ for $n + \lfloor \tau / \lceil \frac{L}{l_r} \rceil \rfloor$ barriers. Since $\lfloor \tau / \lceil \frac{L}{l_r} \rceil \rfloor$ is a constant, the optimal algorithm can solve the *Max-Num-Barrier problem* in $O(n^3)$. □

6.2 Greedy Algorithm

We also propose a faster greedy algorithm for the *Max-Num-Barrier problem*. The basic idea is to repeatedly find the shortest path on the WBG until the deployed number of mobile sensors is reached or no path can be found or the latest found path is longer than $\lceil \frac{L}{l_r} \rceil$. In the end, if some mobile sensors are left, we use them to construct direct barriers. The greedy algorithm is described as follows:

1) Initialize q with 0, and P_q as an empty set.
2) If there exist paths from s to t on the WBG, find the shortest path p using Dijkstra's algorithm; otherwise, go to 5).
3) If the found shortest path is longer than $\lceil \frac{L}{l_r} \rceil$, discard the path, go to 5); otherwise, go to 4).
4) If $|P_q| + |p| < \tau$, remove all the vertices (except for s and t) on the path p from the WBG, put p into P_q and increase q by 1, go to 2). If $|P_q| + |p| = \tau$, $k = q + 1$, stop; otherwise, $k = q$, stop.
5) The maximum number of barriers is $q + \lfloor (\tau - |P_q|)/\lceil \frac{L}{l_r} \rceil \rfloor$.

The pseudocode of the greedy algorithm is presented in Algorithm 3.

Algorithm 3 Max-Num-Barrier-Greedy algorithm

Input: Weighted barrier graph G, L, l_r and τ
Output: N_b

$q \leftarrow 0$ and $P_q \leftarrow \emptyset$
while true **do**
  if there exist paths from s to t on G **then**
    $p \leftarrow$ Dijkstra(G)
    if $|p| \le \lceil \frac{L}{l_r} \rceil$ **then**
      if $|P_q| + |p| \le \tau$ **then**
        $q \leftarrow q + 1$
        $P_q \leftarrow P_{q-1} \cup p$
        update G by removing all the edges incident to the vertices (except s and t) on p
      else
        if $|P_q| + |p| = \tau$ **then**
          $N_b \leftarrow q + 1$, break.
        else
          $N_b \leftarrow q$, break.
    else
      $N_b \leftarrow q + \lfloor (\tau - |P_q|)/\lceil \frac{L}{l_r} \rceil \rfloor$, break.
  else
    $N_b \leftarrow q + \lfloor (\tau - |P_q|)/\lceil \frac{L}{l_r} \rceil \rfloor$, break.

Theorem 11. *The maximum number of barriers found by the greedy algorithm is lower bounded by $\lfloor \tau / \lceil \frac{L}{l_r} \rceil \rfloor$ and upper bounded by $n + \lfloor \tau / \lceil \frac{L}{l_r} \rceil \rfloor$.*

Proof: The proof is similar to that of Theorem 9. □

Theorem 12. *For a deployed sensor network with n stationary sensors and τ mobile sensors, the greedy algorithm can solve the* Max-Num-Barrier problem *in $O(n^3)$.*

Proof: The running time of Dijkstra's algorithm is $O(n^2)$. In the worst case, the greedy algorithm would perform Dijkstra's algorithm $n + \lfloor \tau / \lceil \frac{L}{l_r} \rceil \rfloor$ times. Therefore, the running time for the greedy algorithm is $O(n^3 + n^2 \lfloor \tau / \lceil \frac{L}{l_r} \rceil \rfloor)$. Since $\lfloor \tau / \lceil \frac{L}{l_r} \rceil \rfloor$ is a constant, the greedy algorithm can solve the *Max-Num-barrier problem* in $O(n^3)$. □

7 MCBF PROBLEM

In order to form k barriers, mobile sensors should move to fill in the gaps on the paths. We assume that the moving cost is proportional to the moving distance. Hence, the objective of the MCBF problem is to minimize the total moving distance of mobile sensors to form k barriers. However, the problem is difficult to solve due to the complexity of deploying sensors to fill in a gap.

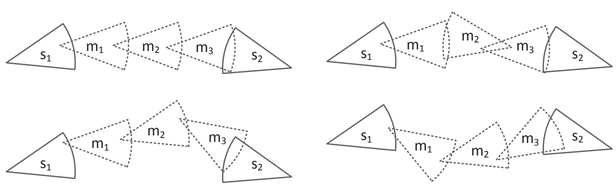

Fig. 6: Illustration of the complexity of deploying mobile sensors to fill in a gap. s_1 and s_2 are two stationary sensors, m_1, m_2 and m_3 are three mobile sensors.

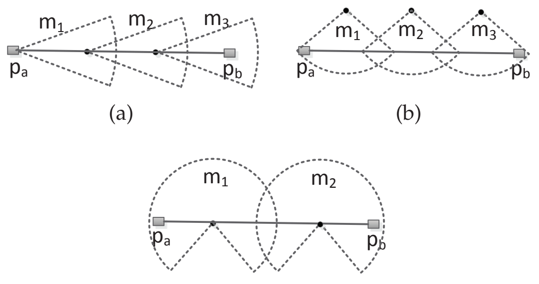

Fig. 7: Three different deployment cases: (a) $l_r = r$ when $2\alpha < \pi$; (b) $l_r = 2r \sin\alpha$ when $2\alpha < \pi$; (c) $l_r = 2r$ when $2\alpha \geq \pi$.

As shown in Figure 6, there are too many ways to fill in a gap. To the best of our knowledge, there is no optimal solution for the MCBF problem. In order to efficiently solve this problem, we divide it into two subproblems. First, how to calculate the target locations for mobile sensors to fill in a gap? Second, how to move mobile sensors to the set of target locations with the minimum total moving distance? In the following, we will describe how to solve these two subproblems to yield a suboptimal solution to the MCBF problem.

7.1 Target Locations Calculation

The target locations are the places where mobile sensors should move to so that k barriers can be formed. Without loss of generality, we consider the calculation of target locations for strong barrier coverage. The calculation for weak barrier coverage is simply a special case where only the x-coordinates of target locations for strong barrier coverage should be considered.

Given two stationary sensors s_a and s_b, suppose the edge $e(s_a, s_b)$ is on one path of the set of k vertex-disjoint paths and its weight $w(s_a, s_b)$ is not zero. Therefore, $w(s_a, s_b)$ mobile sensors should move to $w(s_a, s_b)$ target locations to connect s_a and s_b. Suppose the closest pair of points between s_a and s_b are $p_a = (x_a, y_a)$ on s_a and $p_b = (x_b, y_b)$ on s_b. The minimum distance between s_a and s_b is

$$d_s(s_a, s_b) = d(p_a, p_b) = \sqrt{(x_b - x_a)^2 + (y_b - y_a)^2}$$

Then the minimum number of mobile sensors needed to fill the gap is $w(s_a, s_b) = \lceil \frac{d_s(s_a, s_b)}{l_r} \rceil$. We distribute mobile sensors evenly with the longest line of the sensing sector along the line segment $p_a p_b$. Note that there are too many ways to deploy mobile sensors to fill in a gap. Our deployment provides one the easier ways to calculate the target locations. Therefore, the interval between the two mobile sensors is $d_v = \frac{d_s(s_a, s_b)}{w(s_a, s_b)}$. As mentioned in Section 3, the longest line of a sector could either be the radius or the longest chord when $0 \leq 2\alpha < \pi$, or $2r$ when $\pi \leq 2\alpha \leq 2\pi$. Corresponding to these three cases, we have three deployment strategies, as shown in Figure 7.

Let φ denote the direction of $\overrightarrow{p_a p_b}$. Let h denote the height from the center to the longest chord of a sector. Suppose the target locations are $t_i = (t_i^x, t_i^y, t_i^o)$ for $i = 1, 2, \cdots, w(s_a, s_b)$, where t_i^x and t_i^y are the x-coordinate and y-coordinate of the target location t_i, and t_i^o is the facing direction of the mobile sensor on t_i.

As show in Figure 7(a), when $l_r = r$ and the sensing angle $2\alpha < \pi$, mobile sensors are evenly deployed with the radius along the facing direction on the line segment. Therefore, we have

$$\begin{aligned} t_i^x &= x_a + (i-1) d_v \cos\varphi \\ t_i^y &= y_a + (i-1) d_v \sin\varphi \\ t_i^o &= \varphi \end{aligned}$$

As shown in Figure 7(b), when $l_r = 2r \sin\alpha$ and $2\alpha < \pi$, mobile sensors are evenly deployed with the longest chord on the line segment. Therefore, we have

$$\begin{aligned} t_i^x &= x_a + (i-1) d_v \cos\varphi + \bar{l} \cos(\varphi + \lambda) \\ t_i^y &= y_a + (i-1) d_v \sin\varphi + \bar{l} \sin(\varphi + \lambda) \\ t_i^o &= (\varphi + 3\pi/2) \mod 2\pi \end{aligned}$$

where $\bar{l} = \sqrt{h^2 + (d_v/2)^2}$, $\lambda = \arctan(2h/d_v)$.

Finally, as shown in Figure 7(c), when $l_r = 2r$ and $2\alpha \geq \pi$, mobile sensors are evenly deployed with the diameter on the line segment. Therefore, we have

$$\begin{aligned} t_i^x &= x_a + (i-0.5) d_v \cos\varphi \\ t_i^y &= y_a + (i-0.5) d_v \sin\varphi \\ t_i^o &= (\varphi + \pi/2) \mod 2\pi \end{aligned}$$

7.2 Minimum Total Cost Sensor Movement

Suppose the minimum number of mobile sensors required to form k-barrier coverage is μ. Then μ mobile sensors should move to μ target locations to form k-barrier coverage. Suppose the deployed number of mobile sensors is τ ($\tau \geq \mu$). Let δ_{ij} denote a decision variable, where $\delta_{ij} = 1$ if mobile sensor m_i moves to target location t_j, $\delta_{ij} = 0$ otherwise. d_{ij} is the distance for mobile sensor m_i to move to target location t_j. The objective is to select μ out of τ mobile sensors and move them to μ target locations while minimizing the total moving distance.

$$\text{Minimize} \quad \sum_{i=1}^{\tau} \sum_{j=1}^{\mu} d_{ij} \delta_{ij} \tag{5}$$

$$\text{subject to} \quad \sum_{i} \delta_{ij} = 1, \forall j = 1, 2, \cdots, \mu. \tag{6}$$

$$\sum_{j} \delta_{ij} \leq 1, \forall i = 1, 2, \cdots, \tau. \tag{7}$$

$$\delta_{ij} = 0 \text{ or } 1, i = 1, 2, \cdots, \tau; j = 1, 2, \cdots, \mu.$$

The objective function minimizes the total moving distance with the first constraint indicating that every target location must be assigned with only one mobile sensor and the second constraint indicating that each mobile sensor can be assigned to at most one target location.

Provided with a set of target locations and a set of mobile sensors, the formulated problem indeed is a minimum cost bipartite assignment problem. The Hungarian algorithm [10] [13] provides the optimal solution to this problem and its complexity is proved to be $O(\mu^2\tau)$. Note that [3] studied a similar problem and also used the Hungarian algorithm to solve it. Please refer to [10] [13] for the details of the Hungarian algorithm.

8 Performance Evaluation

In this section, we conduct simulations using Matlab to evaluate the performance of the proposed algorithms.

8.1 The Min-Num-Mobile(k) Problem

The ROI is a belt region of length $L = 500m$ and width $H = 100m$. Initially, stationary sensors are uniformly deployed in the belt region. After the minimum number of mobile sensors is calculated, mobile sensors are deployed uniformly in the belt region and then assigned to different target locations using the Hungarian algorithm to form k-barrier coverage.

The evaluation mainly focuses on four performance metrics:

- The minimum number of mobile sensors required to form k-barrier coverage
- The total moving distance for mobile sensors to form k-barrier coverage
- The average moving distance for mobile sensors to form k-barrier coverage
- The number of direct barriers needed in k barriers

Evaluation of these performance metrics is conducted on different parameters, such as the number of barriers, the number of stationary sensors, the sensing range and the sensing angle (or field of view). For all the simulation results presented in this paper, each data point is an average of 100 experiments. Both weak and strong barrier coverage are studied.

8.1.1 *Effects of the Number of Barriers*

We first evaluate the performance of the algorithms on the number of barriers. Figure 8 shows the experimental results. The number of mobile sensors required to form k-barrier coverage increases as k increases for all the algorithms, as shown in Figure 8(a). The optimal algorithms always use less number of mobile sensors to realize k weak/strong barrier coverage than the greedy algorithms. When $k \leq 3$, the two algorithms give the same result. Therefore, when smaller number of barriers is needed to be formed, greedy algorithms are more suitable because they are faster than the optimal algorithms. We can also observe that forming

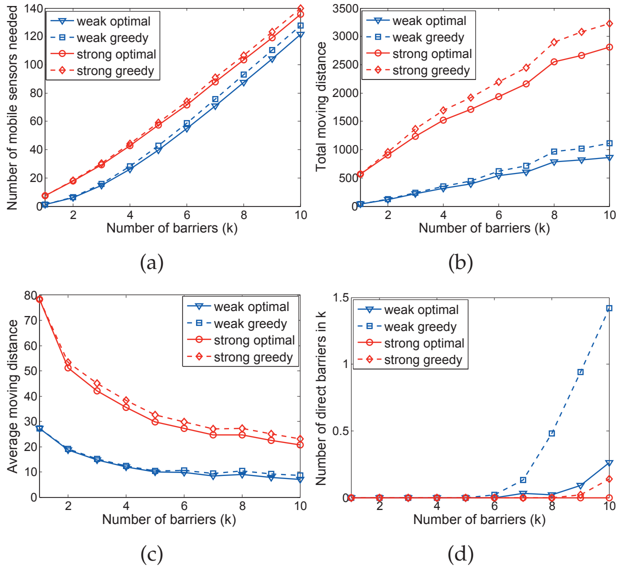

Fig. 8: Performance evaluation on different number of barriers (k), and fixed $n = 100$, $r = 20$ and $\alpha = \pi/4$

strong barrier coverage always requires more mobile sensors than forming weak barrier coverage.

The total moving distance, as shown in Figure 8(b), increases as k increases because more mobile sensors are needed to fill in more gaps when k becomes larger. The total moving distance for strong barrier coverage is always longer than that for weak barrier coverage. This is due to two reasons. First, forming strong barrier coverage requires more number of mobile sensors than forming weak barrier coverage. Second, mobile sensors only need to move in the horizontal direction for weak barrier coverage while they need to move in two dimensions for strong barrier coverage. Although the total moving distance is increasing, the average moving distance for mobile sensors, as shown in Figure 8(c), decreases when k becomes larger. This is because less number of mobile sensors is required for smaller number of barriers, which makes the distribution of mobile sensors less dense. Therefore, each mobile sensor under larger k moves less on average to reach a target location.

As shown in Figure 8(d), no direct barriers are needed when $k \leq 5$, and then the number of direct barriers increases linearly as k increases. This is because stationary sensors can work with mobile sensors to construct barriers when k is small. When most of stationary sensors are used up, if we want to form more barriers, the direct barrier is obviously the best choice.

8.1.2 *Effects of the Number of Stationary Sensors, the Sensing Range and the Sensing Angle*

We then evaluate the effects of the number of stationary sensors, the sensing range and the sensing angle on the performance metrics and show their performance results in Figures 9, 10 and 11, respectively.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication.

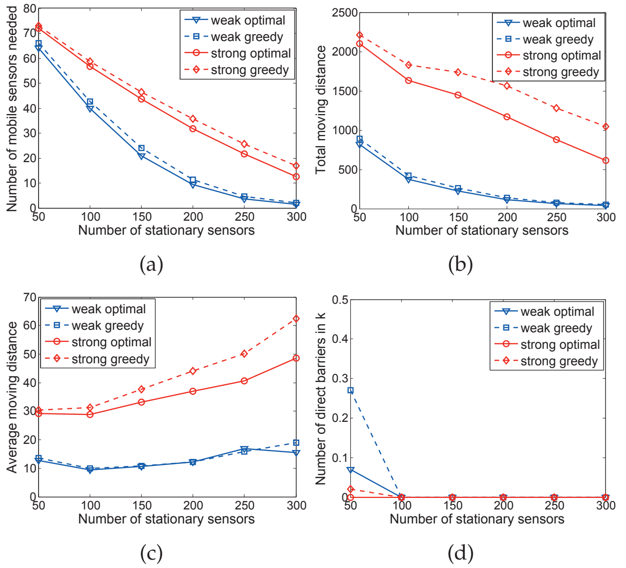

Fig. 9: Performance evaluation on different number of stationary sensors (n), and fixed $k = 5$, $r = 20$ and $\alpha = \pi/4$

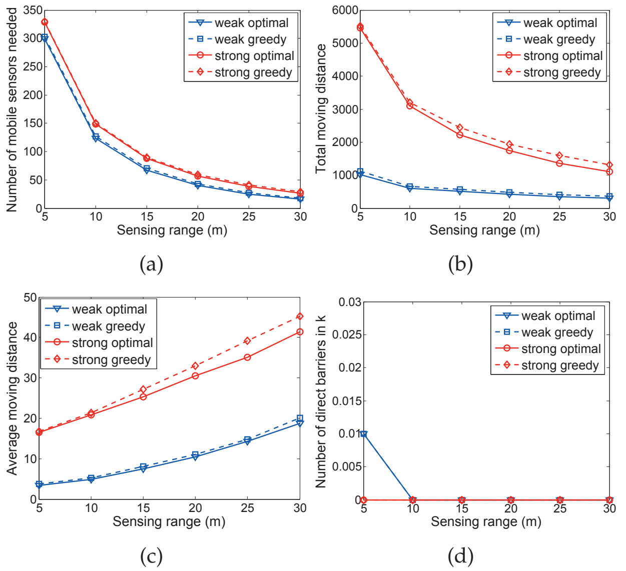

Fig. 10: Performance evaluation on different sensing range (r), and fixed $k = 5$, $n = 100$ and $\alpha = \pi/4$

We can observe that, given a fixed number of barriers to be formed, the number of mobile sensors required decreases when any of these three factors increases. This is because increasing any of them can reduce the number of gaps and the size of gaps between stationary sensors, which then reduces the number of mobile sensors needed. We can also observe that the optimal algorithms always require less mobile sensors than the greedy algorithms, and forming k strong barrier coverage requires more mobile sensors than forming k weak barrier coverage.

The total moving distance, as shown in Figures 9(b), 10(b) and 11(b) decreases as any of these three factors increases. This is because less number of mobile sensors is required to form barriers. The total moving distance for strong barrier coverage is always larger than that of weak barrier coverage. We also observe that the average moving distance of mobile sensors increases as any of these three factors increases. This is because the distribution of mobile sensors are less dense for less number of mobile sensors, which requires mobile sensors to move longer on average to reach target locations.

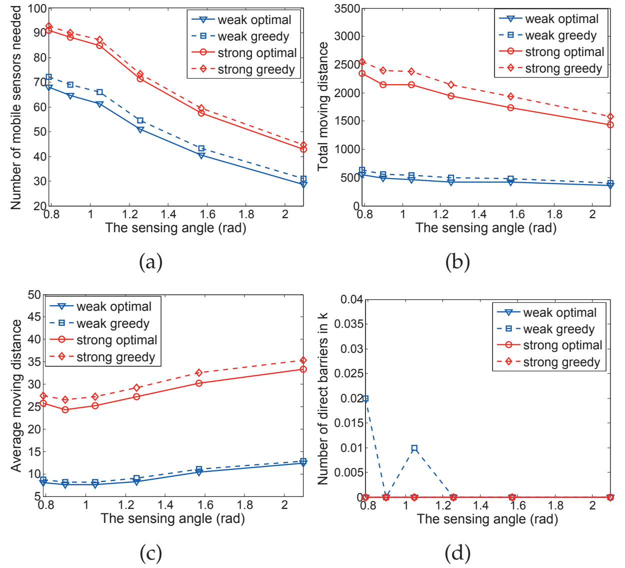

Fig. 11: Performance evaluation on different sensing angle (2α), and fixed $k = 5$, $n = 100$ and $r = 20$

As shown in Figures 9(d), 10(d) and 11(d), when any of these three factors is small, direct barriers may be needed. This is because almost all stationary sensors have been used up before reaching the specified number of barriers. Then the only way to achieve the specified number of barriers is to use direct barriers. We also observe that forming k weak barriers requires more direct barriers than forming k strong barriers. This is because forming weak barriers used more stationary sensors than forming the same number of strong barriers. In other words, stationary sensors could be more easily used up for weak barrier coverage. Therefore, more direct barriers are involved to form k weak barriers than those for k strong barriers.

8.2 The Max-Num-Barrier Problem

In this section, we evaluate the performance of the proposed algorithms for the *Max-Num-Barrier problem*.

The ROI is a belt region of length $L = 500m$ and width $H = 100m$. Initially, both n stationary sensors and τ mobile sensors are uniformly deployed in the ROI. After the maximum number of barriers and the set of barriers are found, mobile sensors can move to target locations to form multiple barriers. The maximum number of barriers is the performance metric of the evaluation. The evaluation is conducted

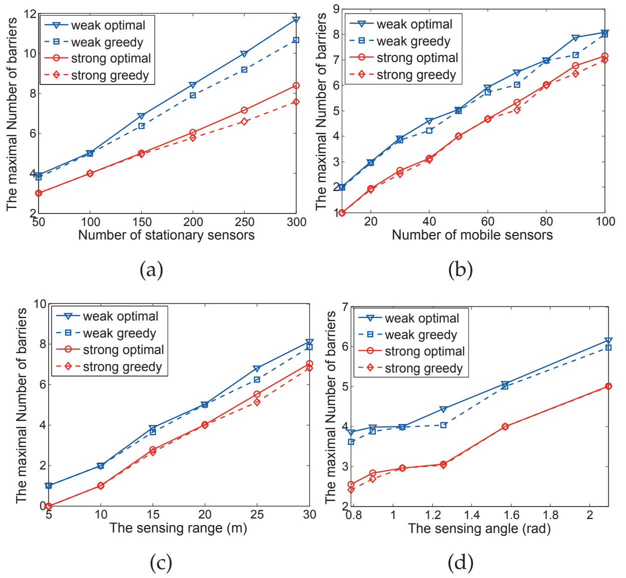

Fig. 12: Performance evaluation on: (a) different number of stationary sensors, fixed $\tau = 50$, $r = 20m$ and $\alpha = \pi/4$; (b) different number of mobile sensors, fixed $n = 100$, $r = 20m$ and $\alpha = \pi/4$; (c) different sensing range, fixed $n = 100$, $\tau = 50$ and $\alpha = \pi/4$; (d) different sensing angle, fixed $n = 100$, $\tau = 50$ and $r = 20m$

on different parameters including the number of stationary sensors, the number of mobile sensors, the sensing range and the sensing angle.

Figure 12 shows the performance evaluation results. We can observe that the increasing of any one of the four factors could result in the increasing of the maximum number of barriers. The proposed optimal algorithms always perform better than the greedy algorithms. However, the difference between them is not very obvious. For example, the maximum number of barriers for greedy and optimal algorithm are almost the same in Figure 12(b). We also observe that the maximum number of weak barriers is always larger than that of strong barriers, given the same network deployment. That is, it is much easier to form weak barriers than to form strong barriers.

8.3 Computation Time Comparison

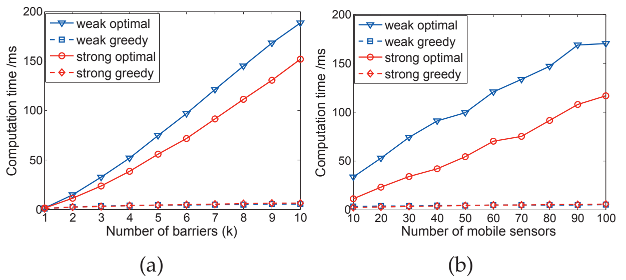

Fig. 13: Computation time comparison between the optimal and greedy algorithm for: (a) Min-Num-Mobile(k) problem; (b) Max-Num-Barrier problem with fixed $n = 100$, $r = 20$ and $\alpha = \pi/4$.

Figure 13 demonstrates the comparison of computation times between the optimal algorithms and the greedy algorithms. The algorithms run on Thinkpad T420 with CPU of 2.80GHz and 4GB RAM. We can see that the computation times of the optimal algorithms increase significantly with the increase of the number of barriers or the number of mobile sensors deployed. The computation times of the greedy algorithms, however, do not increase significantly. Therefore, although two algorithms for the same problem have the same running time in the worst case, the greedy algorithm is usually faster and more scalable to large-scale networks as compared to the optimal algorithm.

9 Discussion

In this paper, we have proposed algorithms for the barrier coverage formation problem under the assumption that the sensor network is composed of the same type of sensors. In reality, it is more than likely that different types of sensors may coexist in an ROI for intruder detection. It is worth noting that the WBG model and the proposed algorithms would also work for sensor networks with different types of sensors.

Let us consider a heterogeneous network where there are different types of stationary and mobile sensors. Because different types of sensors have different hardware costs, the objective becomes minimizing the total cost of mobile sensors needed to form k-barrier coverage. By letting the weight of each edge as the minimum cost of mobile sensors needed to connect two stationary sensors, we can construct the WBG for the network. Once the WBG is constructed, we can apply the optimal algorithm to find the minimum cost of mobile sensors needed to form k-barrier coverage.

Also note that some assumptions in this paper may not be very realistic. For example, in reality, the ROI may be very complicated. It is possible that sensors do not know their accurate locations and obstacles may exist in the sensing regions of sensors. These conditions may be interpreted as faults that would affect the performance of barrier coverage and the design of fault tolerant algorithm for barrier coverage formation would be one of our future work.

10 Conclusions

In this paper, we studied the barrier coverage formation problem in hybrid directional sensor networks. We introduced a novel weighted barrier graph model for the barrier coverage formation problem. We proposed an optimal algorithm and a faster greedy algorithm to find the minimum number of mobile sensors required to form k-barrier coverage with pre-existing stationary sensors. We also proposed an optimal algorithm and a faster greedy algorithm to determine the maximum number of barriers when both the stationary and mobile sensors have been pre-deployed. In addition, we formulated the problem of moving mobile sensors to form k-barrier coverage with the minimum total moving distance as a minimum cost bipartite assignment problem and solved it using the Hungarian algorithm. Both analytical and experimental studies demonstrated the effectiveness of our proposed algorithms.

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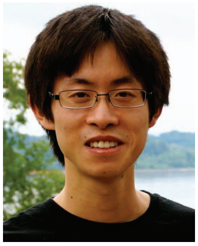

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